Sun Setting over the Menkaure Pyramid during Summer Solstice
The Giza Pyramid Triangle Incenter Observation Point

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Abstract

The scope of this paper is to show that the placement of the three large pyramids of Giza, as also the height of the last one erected, was planned so that when a person stands in the triangle incenter formed by the center base position of these three pyramids, then given the date is the Summer Solstice of the archeologically accepted epoch that the last pyramid was completed, he would watch in the afternoon the Sun set exactly over the apex of this (Menkaure's) pyramid. In this paper using the latest geodetic data presented by Glen Dash, we investigate the accuracy of this scenario, taking into account among other things, local altitude data, horizontal dip, terrestrial and astronomical refraction, as also Earth obliquity and nutation periodicity. The results are compared to relative calculations performed with the assistance of popular astronomy software. We also compute the solar alignment divergence in our epoch.

Keywords

Giza, Pyramids, Menkaure, Fourth Dynasty, Sun, Summer Solstice, Ra, Circumcircle, Incenter

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1. Introduction: Giza Planning

Many people have asked what the true function of the Egyptian pyramids was. Although Egyptologists propose that they were used as tombs, researchers have wondered whether there are geometric or astronomical factors behind their design. Looking into the ground plan of the three large Giza pyramids, I remember being amazed about the fact that if we draw a circle that crosses over the apex of each one of these pyramids(Khufu's, Khafre's, and Menkaure's) we notice that the diameter of the circle is very close to 9,000 ancient Egyptian royal cubits. To compute the radius in this case I used data from archeologist Flinders Petrie's 1883 survey.

Recently, Rodney Hale and Andrew Collins published an article titled "A Study of the Simple Geometrical Relationship of the Main Monuments of Giza and a Possible Connection to Stars". There they define a Convergence Viewpoint on the extension of the Khafre pyramid center base to pyramid circle center curve(an extension of the circle radius). At the position they define, they propose that all three Cygnus wings stars (δ Cyg, γ Cyg, and ε Cyg) align to each one of the three Giza pyramid apex's(Khufu's, Khafre's, and Menkaure's). An interesting idea, but this extension is still arbitrary in nature, chosen so that the stars match the pyramid directions.

1.1 The Hypothesis

I started studying the center circle position to see if other alignments could have dictated the planning of the Giza pyramid layout. In this article I will show that if one stands in the center of the circle that crosses over the horizontal position of each one of the three pyramid apex's at Giza(or equivalently their center base points), during the Summer Solstice, the year - or epoch - that the last pyramid, that of Menkaure was completed, then he would notice that the center of the circular Sun disk, would align with the original "geometric" apex of Menkaure's pyramid, just before dusk.

The summer solstice occurs when the tilt of a planet's semi-axis, in either northern or southern hemispheres, is most inclined toward the star that it orbits. Earth's maximum axial tilt toward the Sun is 23° 26'. This happens twice each year, at which times the Sun reaches its highest position in the sky as seen from the north or the south pole. The summer solstice occurs during a hemisphere's summer. This is the northern solstice in the northern hemisphere and the southern solstice in the southern hemisphere. Depending on the shift of the calendar, the summer solstice occurs sometime between June 20 and June 22 in the northern hemisphere. The term is colloquially used like midsummer to refer to the day on which solstice occurs.

Let's see what we'll need to prove our hypothesis. First of all let us define the length of the royal cubit. The ancient Egyptian royal cubit (meh niswt) is the earliest attested standard measure. Cubit rods were used for the measurement of length. A number of these rods have survived. Two are known from the tomb of Maya, the treasurer of Tutankhamen, in Saqqara, while another was found in the tomb of Kha in Thebes. Fourteen such rods, including one double cubit rod, were described and compared by Lepsius in 1865. These cubit rods range from 523 to 529 mm in length. In Glen Dash's article "The Great Pyramid's Footprint: Results from Our Survey" we are informed that the original mean Khufu pyramid base length was 230.363 meters. Flinders Petrie writes "If a strictly weighted mean be taken it yields 20.620" ± .004". Agreeing that the architects planned the Khufu pyramid base length to be exactly 440 royal cubits, we get:

c = 230.363 m / 440 = 0.5235523 m

1.2 The Giza Circle

We will now use the data from Glen Dash's article "Where, Precisely, are the Three Pyramids of Giza?" so as to compute the radius of the circle, and to depict the center circle position as also the center base of the three pyramids through Google Earth Pro 7. In geometry, the incircle or inscribed circle or circumcircle of a triangle is the largest circle contained in the triangle. It touches (is tangent to) the three sides. The center of the incircle is a triangle center called the triangle's incenter.

In Figure 4 - page 15 of Dash's article the locations of the Pyramids of Giza are depicted on the GPMP grid. We read that the locations were derived by the author using data from the surveys of Petrie and Goodman/Lehner. From this data we will compute the distance between the center base of Khafre's pyramid and Khufu's pyramid which we will name x_{12}. We will also compute the distance between the center base of Khafre's pyramid and Menkaure's pyramid which we will name x_{23}. Finally, we will name the distance between the center base of Khufu's pyramid and Menkaure's pyramid x_{13}.

a_{12} = West - East Khufu to Khafre center = 500,000 m - 499,666.1 m = 333.9 m

b_{12} = North - South Khufu to Khafre center = 100,000 m - 99,645.7 m = 354.3 m

With the help of the Pythagorean theorem, the hypotenuse of a right triangle has a length x_{12} of:

$$x_{12} = \sqrt{a_{12}^2 + b_{12}^2} = \sqrt{333.9^2 + 354.3^2}\, m = 486.8446 \text{ m}$$

Working in similar fashion:

a_{23} = West - East Khafre to Menkaure center = 499,666.1 m - 499,426.6 m = 239.5 m

b_{23} = North - South Khafre to Menkaure center = 99,645.7 m - 99,260.0 m = 385.7 m

The distance x_{23} is then computed as follows:

$$x_{23} = \sqrt{a_{23}^2 + b_{23}^2} = \sqrt{239.5^2 + 385.7^2}\, m = 454.0096 \text{ m}$$

And for the last pyramid distance x_{13}:

a_{13} = West - East Khufu to Menkaure center = 500,000 m - 499,426.6 m = 573.4 m

b_{13} = North - South Khufu to Menkaure center = 100,000 m - 99,260.0 m = 740.0 m

The distance x_{13} is then computed as follows:

$$x_{13} = \sqrt{a_{13}^2 + b_{13}^2} = \sqrt{573.4^2 + 740.0^2}\, m = 936.1557\ m$$

The distance therefore between the Khufu and Khafre pyramid center bases is 486.8446 m, the distance between the Khafre and Menkaure pyramid center bases is 454.0096 m, and the distance between the Khufu and Menkaure pyramid center bases is 936.1557 m.

The azimuth between Khafre's center base and Khufu's center base φ_{12}, as also the azimuth between Khafre's center base and Menkaure's center base φ_{23} is also needed. An azimuth is an angular measurement in a spherical coordinate system. The vector from an observer to a point of interest is projected perpendicularly onto a reference plane; the angle between the projected vector and a reference vector on the reference plane is called the azimuth. Usually, the azimuth of true North is 0°, the azimuth of true East is 90°, the azimuth of true South is 180°, and the azimuth of true West is 270°. We can now compute φ_{12} trigonometrically using the previous data:

φ_{12} = tan^{-1}(a_{12}/b_{12}) = tan^{-1}{(333.9 m)/(354.3 m)} = 43.3021°

The azimuth therefore between the center base of Khafre's pyramid and that of Khufu is 43.3021°. Note that the North-East direction is 45°. Continuing with azimuth φ_{23}:

φ_{23} = 180° + tan^{-1}(a_{23}/b_{23}) = tan^{-1}{(239.5 m)/(385.7 m)} = 180° + 31.8382° = 211.8382°

The azimuth therefore between the center base of Khafre's pyramid and that of Menkaure is 211.8382°. Note that the South-West direction is 225°.

With this information at hand we use Google Earth software to determine the geographical coordinates of the center base of Khafre's pyramid. The exactness of these coordinates are not important here because we can always calibrate the resulting incenter coordinates to any difference in latitude or longitude - see the end of this article. We therefore use a latitude of 29.975987° North, and a longitude of 31.130731° East, for Khafre's center base position. Using an internet tool included in the sources section we calculate the destination geographic coordinates based on the distance, bearing and starting geographic coordinate, for the WGS84 Earth ellipsoid model.

The starting geographic coordinates are that of Khafre's center base. We start by targeting Khufu's center base so we use the distance x_{12} and the azimuth φ_{12}. Inserting this data we determine that the Khufu center base coordinates are 29.979183° North and 31.134191° East. In similar fashion, we target Menkaure's center base so we use the distance x_{23} and the azimuth φ_{23}. Inserting this data we determine that the Menkaure's center base coordinates are 29.972508° North and 31.128249° East.

Since the three sides x_{12}, x_{23}, and x_{13} of the triangle are on a plane we can use the following equation to compute the radius R of the circumcircle.

$$R = \frac{x_{12} \cdot x_{13} \cdot x_{23}}{\sqrt{(x_{12}+x_{23}+x_{13})(x_{12}+x_{23}-x_{13})(x_{12}+x_{13}-x_{23})(x_{13}+x_{23}-x_{12})}} = 2{,}355.0959\ m$$

Converting to royal cubits we find:

R = 2,355.0959 m = 2,355.0959 / 0.5235523 royal cubits = 4,498.3 royal cubits

The diameter of the circumcircle D is therefore double this length:

D = 2 × R = 2× 4,498.3 royal cubits = 8,996.6 royal cubits

The diameter of the circumcircle is 3.4 units short of 9,000 royal cubits.

Using the ruler tool in Google Earth we find the point that is equidistant to the outer pyramid center bases(that of Khufu and Menkaure), which at the same time has a distance from each one equal to the radius length(2,355.096 m). The geographic coordinates of this point, which is the triangle's incenter are 29.963089° North, and 31.150121° East.

2. Where is the apex?

With the help of the same online tool we calculate the great circle bearing(azimuth) between the incenter and the center base of Menkaure's pyramid. We name this azimuth φ_{O3}:

φ_{O3} = 296.32244° = 296° 19' 21"

On the top of this page we provide a Google Earth photo of the Giza pyramids along with the incenter(satellite image credit: Google Earth/Digital Globe, 2016).

Using the terrain tool of Google Earth we determine that the altitude is 57 feet or 17.374 meters. The average male eye height is around 1.63 meters. With this information we can compute the observational altitude h_O for the observer standing on the incenter:

h_O = 17.374 m + 1.63 m = 19.0036 m

2.1 Altitude calculation

According to archeologists V. Maragioglio and C. Rinaldi the base of Menkaure's pyramid is 71.42 m above sea level. According to Petrie, the height of Menkaure's pyramid is 65.1256 m(2,564" ± 15") or 2,580.8" ± 2.0" by the granite courses. Using the first figure leads to a pyramid apex altitude h_M of 136.5456 meters above sea level. This is also close to archeologist Mark Lehner's value for the height(65 m).

h_M = 65.1256 m + 71.42 m = 136.5456 m

We compute the altitude difference Δh between the observation point and the target point(pyramid apex):

Δh = h_M - h_O = 136.5456 m - 19.0036 m = 117.542 m

Since the Earth is not a flat plane, in order to compute the length of the direct line between the observer and the target, we first need to calculate the dip due to Earth's curvature. If we consider a perfect spherical Earth, then the Earth's radius is vertical to the horizon line at the observation point. We have a right triangle formed between the center of the Earth, the observation point, and the point that the radius of the Earth, toward the target, intersects the horizon of the observation point(see Figure 1). We thus need to compute the angle at the center of the Earth. The radius of the Earth changes depending on the latitude. The distance from the Earth's center to a point on the spheroid surface at geodetic latitude φ is computed as follows:

$$R_\varphi = R(\varphi) = \sqrt{\frac{(a^2\cos{(\varphi)})^2+(b^2\sin{(\varphi)})^2}{(\alpha \cdot \cos{(\varphi)})^2+(b \cdot \sin{(\varphi)})^2}}$$

where α and b are, respectively, the equatorial radius and the polar radius. By inserting the equatorial radius (6,378,137 m), the polar radius (6,356,752.3 m), and the latitude of the incenter (29.963089°) we are led to a radius value of:

R_φ = 6,372,836.293 meters

The distance from the center of the Earth to the observation point R_e is equal to the sum of radius R_φ plus the altitude above sea level of the observation point h_O:

R_e = R_φ + h_O = 6,372,836.293 m + 19.0036 m = 6,372,855.296 m

We use the radius of the circle and this radius R_e to compute the Earth center angle θ_E:

θ_E = 360° × R / (R_e × 2 × π) = 360° × 2,355.0959 m /(6,372,855 m × 2 × π) = 0.021174°

The dip t is then computed to be:

t = R_e×{1-cos(θ_E)} = (6,372,855 m) ×{1 - cos(0.021174°)} = 0.4352 m

The altitude difference Δh' between the observation point and the pyramid apex taking into account the dip, due to the curvature of the Earth is then:

Δh' = Δh - t = 117.542 m - 0.4352 m = 117.1068 m

Since R is considerably smaller than the perimeter of the Earth we consider it equal to the direct line w that crosses the observation point and the target point radii at equal altitude{sin(θ_E) ≅ 0}. We consider a triangle formed by length w, length Δh, and the angle between them which is equal to one right angle plus θ_E. Using the law of cosines we can then compute the other side of the triangle which is the direct distance f between the observer and the target point.
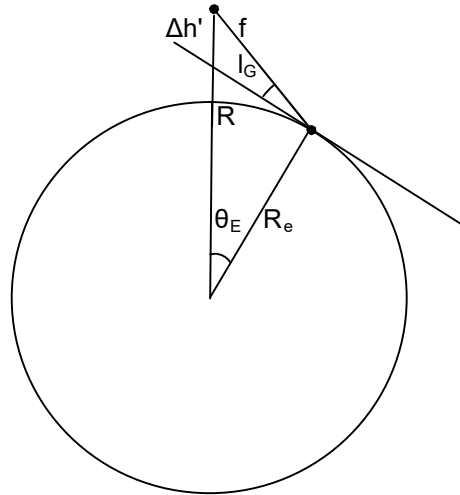
f^2 = w^2 + $Δh^2$ - 2 × w × Δh × cos(90°+ θ_E) ⟹ f = 2,358.049 m

Having computed the direct distance f we can now compute the geometric altitude I_G of the pyramid apex as seen from the observation point. With the help of the law of cosines we get:

$2 \times w \times \Delta h \times \cos(I_G) = f^2 + w^2 - \Delta h^2 \Rightarrow I_G = 2.8466°$

In the following diagram we have drawn a model including the measures mentioned above.

Figure 1

3. Terrestrial Refraction

Atmospheric refraction is the deviation of light or other electromagnetic wave from a straight line as it passes through the atmosphere due to the variation in air density as a function of altitude. This refraction is due to the velocity of light through air decreasing with increased density. Terrestrial refraction deals with the apparent angular position and measured distance of terrestrial bodies. The refraction Ω at the observer measured in arc seconds can be computed using the following equation(Geodesy - Bomford, Guy):

$$\Omega = 16.3 \frac{LP}{T^2} \left(0.0342 + \frac{dT}{dh} \right) cos\beta$$

L is the length of the line of sight so it is equal to f. P is air pressure in millibars, so taking into account the average air pressure of Cairo the last 6 years, at the time the Sun set during Summer Solstice, we set P to 1009 mbar. T is air temperature in degrees Kelvin. Taking into account the average air temperature of Cairo the last 6 years, at the time the Sun set during Summer Solstice(33.5° C), as also the urban heat island affect, leads us to set the temperature to 30° C. β is the angle of the ray to the horizontal so it is equal to I_G. dT/dh is the local vertical temperature gradient, which is also known as the troposphere temperature lapse rate. In our calculation we use the 6.5 K/km value. Using these values we compute the refraction angle Ω to be:

$$\Omega = 16.3 \frac{LP}{T^2} \left(0.0342 + \frac{dT}{dh} \right) cos\beta = 17.15436" = 0.0047651°$$

To compute the apparent altitude I_A of Menkaure's apex as seen from the observation point, all we have to do is add the refraction angle Ω to the geometric altitude I_G:

$I_A = I_G + \Omega = 2.8466° + 0.0047651° = 2.8514° = 2° 51' 5"$

Menkaure, was an ancient Egyptian pharaoh of the 4th dynasty during the Old Kingdom, who is well known under his Hellenized names Mykerinos - Μυκερίνος (by Herodotus) and Menkheres - Μενχέρης (by Manetho). According to Manetho, he was the throne successor of king

Bikheris. Menkaure became famous for his tomb, the Pyramid of Menkaure, at Giza and his beautiful statue triads, showing the king together with his wives Rekhetre and Khamerernebty. He reigned from 18 to 22 years starting at about 2530 BC. He is considered to have died at about 2500 BC.

4. Earth obliquity and nutation

Earth's obliquity oscillates between 22.1° and 24.5° on a 41,000 year cycle. It is currently 23.437 15° and decreasing. To compute the obliquity with high precision over ± several centuries, we use the Astronomical Almanac for 2010. The equation for obliquity ε is as follows:

$$\varepsilon = 23° \, 26' \, 21.406'' - 46.836769'' \, T - 0.0001831'' \, T^2 + 0.00200340'' \, T^3 - 5.76'' \times 10^{-7} \, T^4 - 4.34'' \times 10^{-8} \, T^5$$

where here t is multiples of 10,000 Julian years from J2000.0. If we insert the date July 14, 2501 BC through -44.9947 centuries into the equation we compute the obliquity of the Earth to be ε_P:

$\varepsilon_P = 23.97545°$

Periodic motions of the Moon and of Earth in its orbit cause much smaller short-period oscillations of the rotation axis of Earth, known as nutation, which add a periodic component to Earth's obliquity. The true or instantaneous obliquity includes this nutation. The nutation period is 18.60 years. It is the same as the precession period of the lunar ascending node. Using the nutation calculation website sited in the end of this article we can compute the smallest value of nutation in obliquity taking place on June 17, 2015 AD(-15.3"). If we consider 243 nutation cycles we have a time duration T of:

T = 243 × T_n(yr) = 243 × 18.6 years = 4,519.8 years = 4,520 years - 73 days

Subtracting 4,520 years and adding 73 days to June 17, 2015 AD we end up at August 29, 2506 B.C.. Notice that since there is no 0 AD we subtract 2014 years instead of 2015. At this date therefore based on the presently known nutation period, we would expect to find the smallest (largest negative) value of nutation in obliquity. Since from the largest value to the smallest value we have half a period, then from the smallest value to the value of zero we would have one fourth a period. This time period $T_{1/4}$ is therefore:

$T_{1/4}$ = T_n(yr)/4 = 18.60 years / 4 = 4 years + 237 days

If we take the above date (August 29, 2506 B.C.) and we add this time period ($T_{1/4}$) we would find the date when nutation is zero. This date is April 24, 2501 BC. The closest Summer Solstice took place on July of this year. Also of interest is that Menkaure's reign (between 18 to 22 years) reminds us pretty much of the nutation period (18.6 years).

5. Astronomical Refraction

Before we start incorporating the 2501 BC obliquity of the Earth in our calculations we need to compute the astronomic atmospheric refraction of the Sun rays. G. G. Bennett developed a simple empirical formula for calculating refraction from the apparent altitude h_α which gives the refraction R in arcminutes:

$$R = \cot\left(h_a + \frac{7.31}{h_a + 4.4}\right)$$

According to Meeus, the results of this simple formula are accurate to 0.07′ within the altitude range 0° - 90°. Bennet's formula assumes an atmospheric pressure of 101.0 kPa and a temperature of 10 °C. For different pressure P and temperature T, refraction calculated from these formulas is multiplied by factor k:

$$k = \frac{P}{101} \cdot \frac{283}{273 + T}$$

In our case we equate h_α to the apparent terrestrial altitude l_A we previously computed (2° 51' 5"), we assign 30° C to T, and 100.9 kPa to P. We then calculate the astronomic refraction of the Sun R_S to be:

R_S = 13.83' = 0.23051°

Since due to atmospheric refraction the Sun appears higher in the sky then it is in reality, we subtract this value from the apparent latitude to compute the Sun's true geometric altitude h_G:

$h_G = h_A - R_S$ = 2.8514° - 0.23051° = 2.62088°

6. Solar Azimuth

The solar azimuth angle φ_S can be calculated to a good approximation with the following formula:

$$\varphi_S = cos^{-1} \left(\frac{\sin(\delta) - \cos(\theta_S) \cdot \sin(\Phi)}{\sin(\theta_S) \cdot \cos(\Phi)} \right)$$

δ is the current Sun declination, θ_S is the solar zenith angle, and Φ is the local latitude.

Φ is therefore the latitude of the incenter (29.96309°). Since we are considering the Summer Solstice the Sun declination is equal to the Earth's obliquity ε_P (23.97545°) during the time in question (2501 BC). On the other hand, the solar zenith angle is the angle between the zenith and the centre of the sun's disc. The solar elevation angle h_G is the altitude of the sun, the angle between the horizon and the centre of the sun's disc. These two angles are complementary:

θ_S = 90° - h_G = 90° - 2.62088° = 87.37912°

We can now calculate the solar azimuth angle:

φ_S = 296.30408°

7. Alignment Accuracy

The final azimuth difference Δφ between the observation point to pyramid apex direction φ_{O3}, and the solar azimuth angle φ_S is:

Δφ = φ_S - φ_{O3} = 296.30408° - 296.32244° = -0.0183656° = - 0° 1' 6"

This angle error is more than three times smaller than the orientation error of Khufu's pyramid in regards to the cardinal directions(-3 54"). Taking the triangle incenter we can convert this angle to an equivalent arc of 75 centimeters in length. The accuracy of this alignment is evident if we consider trying to spot a length this small from a distance of about 2.4 kilometers.

8. Using Astronomy Software

In order to verify this alignment we will use Cartes du Ciel Skycharts 3.10 astronomy software. Due to the reason previously stated(nutation) we will use 2501 BC as the observation year. We have set the geographical coordinates of the observation point to that of the circumcircle(29.963089° N, 31.150121° E). The altitude is set to 19 meters(h_O). In the Horizon, atmospheric refraction parameters, the pressure is set to 1009 millibar, the temperature is set to

30° C, the humidity is set to 33%(average of last six years during Sun set at summer solstice at Cairo), and the troposphere temperature lapse rate is set to 6.5 K/km.

On July 13, 2501 BC, the Sun passes the geometric horizon(h = -3.8") having an azimuth of 297° 57' 59.7". On July 14, 2501 BC, the Sun passes the geometric horizon(h = 0.4") having an azimuth of 297° 58' 0.4". Finally, on July 15, 2501 BC, the Sun passes the geometric horizon(h = 3.2") having an azimuth of 297° 57' 30.4". The maximum azimuth is during the Summer Solstice, therefore during the year 2501 BC Summer Solstice took place on July the 14th.

We will now find the time of day that the apparent altitude of the Sun is exactly equal to the apparent altitude of the Menkaure pyramid apex(l_A). At this exact moment we will note the azimuth of the Sun and then compare it to the exact azimuth of Menkaure pyramid's apex that we have already computed(φ_{O3}). We will name the difference between these two azimuths $\Delta\varphi$.

At 18:35:37 EET(UT+02:00) on the 14th of July of 2501 BC, the apparent altitude of the Sun is +02° 51' 00.2" while the geometric altitude of the Sun is +02° 37' 27.3". At this exact instance the Sun's azimuth y_S is as can be seen in the Cartes du Ciel screenshot(next page) +296° 17' 50.8". It's at this instant that the apparent altitude of the Sun, which the software computes having taken into account astronomic refraction, is closest to the apparent altitude of the pyramid apex(2° 51' 5"). We can therefore compute the azimuth difference $\Delta\varphi$ as follows:

$$\Delta\varphi = y_S - \varphi_{O3} = 296° 17' 51” - (296° 19' 21") = - 0° 1' 30" = -0.025°$$

The difference between this value and out own calculation (1' 6") is 24".
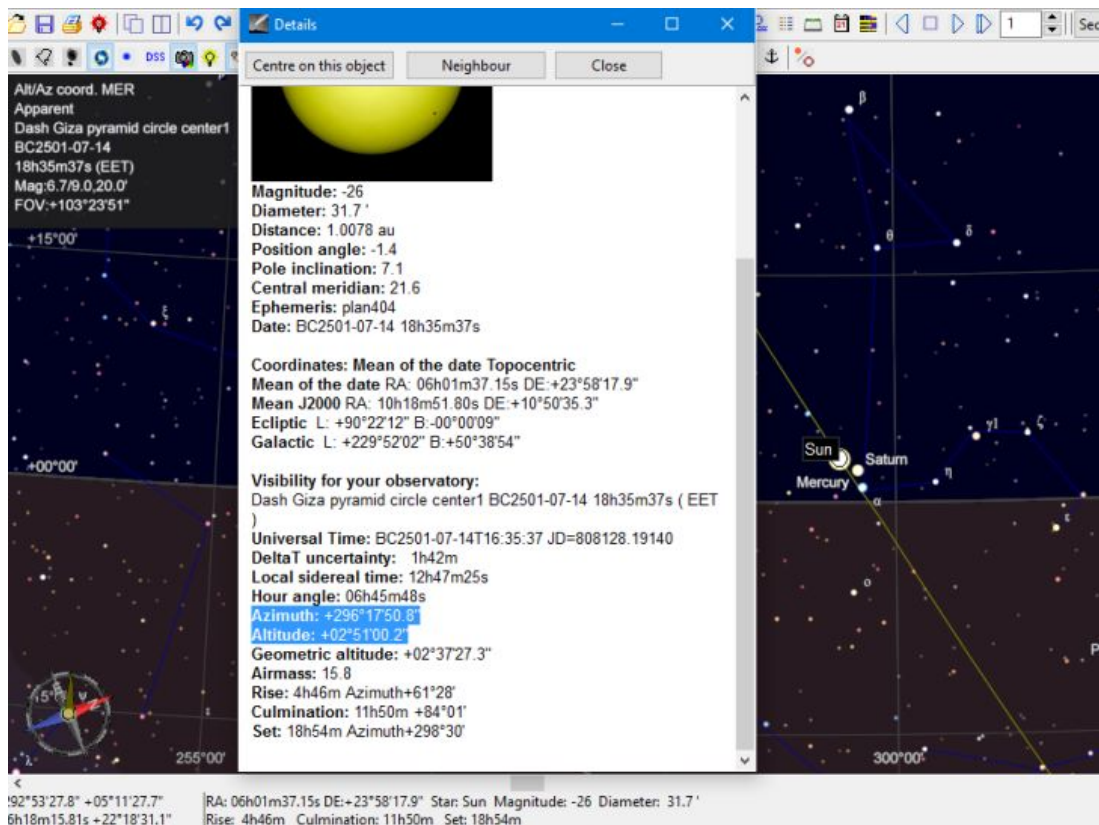
In the above picture as also the one on the next page (image credit: Cartes du Ciel Sky Charts version 3.10) we notice that the position of star Regulus is close to the base and behind Menkaure's pyramid(not far from the right corner). On the other hand planet Saturn visually is

pretty much on the right slope of the pyramid. The center of the Sun disk is off course at the exact pyramid's apex. Naturally since it is day, Regulus and Saturn are not visible.

We can also consider another astronomy program. Using Stellarium 0.14.3, we enter the same coordinates for the observation point and we set the altitude to 19 meters. We set the light pollution to 1, meaning an excellent dark sky site. The air pressure is set to 1009 mbar, the temperature is set to 30° C, and the extinction coefficient to 0.3. The date is once again July 14, 2501 BC(Summer Solstice). According to Stellarium, at 18:32:53 EET the apparent altitude of the Sun is 2.8496° and its azimuth is 296.3015°, or equivalently 296° 18' 5". The error (-1' 16") in this case is 14 arc seconds smaller in relation to the value computed using Cartes du Ciel.

9. Nutation and Archeochronologies

From the web article titled "Precession and Forced Nutation of the Earth" we find an equation for calculating the nutation in obliquity amplitude δφ:

$$\delta\varphi = \frac{3}{2} \cdot \frac{\varepsilon \cdot \iota_m \cdot \mu_m \cdot T_n(yr)}{T_s(day) \cdot [T_m(yr)]^2} \cdot \frac{cos(2 \cdot \theta_0)}{sin(\theta_0)}$$

ε is Earth's flattening(0.0033528), ι_m is the Moon's inclination to the ecliptic(5.145°), μ_m is the Moon to Earth mass ratio(0.0123), $T_n(yr)$ is the nutation period in years(18.6), $T_s(day)$ is the tropical Earth year(365.24219), $T_m(yr)$ is the Moon's (synodic) orbital period in years(0.080852), and θ_0 is Earth's obliquity(23.97545° for 2501 BC). We can thus compute δφ to be:

δφ = 0.0040867° = 14.7"

Based on the nutation period we have a minimum value for the nutation obliquity as also a minimum (negative) value for the solar azimuth angle at the Summer Solstice of 2506 BC. On the other hand we have a maximum (positive) value for the nutation obliquity as also a maximum value for the solar azimuth angle at the Summer Solstice of 2515 BC. If we perform the relative calculations, changing also the obliquity due to the difference in year, we will see that the azimuth difference in regards to the pyramid apex direction is as follows:

δa(2506 BC) = 0° 1' 23"

δa(2515 BC) = 0° 0' 49"

We notice that year 2515 BC the error in azimuth is the smallest.

If we take a half nutation period then the nutation obliquity is close to zero once again. Let's now take 18 ½ nutation periods in the past in relation to April 24 2501 BC. This time period is 158.1 years or 158 years and 37 days. Subtracting therefore this time period from April 24 2501 BC we end up at March 18, 2659 BC. This is more or less the epoch that the oldest pyramid of Egypt, that of Djoser was erected at Saqqara. The time difference between January 1, 2661 BC and January 1, 2000 AD is exactly 4,600 years. Inserting these -46 eons in our Earth obliquity equations yields a value of 23.99104°. This leads to a solar azimuth angle of 296.32243°, which translates to an error in regards to the pyramid apex azimuth of -0.04". This is practically zero.

Here we present a photo (image credit: Cartes du Ciel Sky Charts version 3.10) during the Summer Solstice date of July 15, 2660 BC. We have overlaid a isosceles triangle with the shape of Menkaure's pyramid(right is the North-East vertex and left is the South-West vertex) as we would see it from the circumcircle center when the Sun is at top of the apex. This perfect alignment based on the data we used occurred when the Step Djoser pyramid was erected in Saqqara. We notice the right, North-East pyramid corner aligning perfectly with the position of planet Uranus.


Uranus is the seventh planet from the Sun. It has the third-largest planetary radius and fourth-largest planetary mass in the Solar System. Uranus's apparent magnitude fluctuates between +5.32 and +5.9, placing it just within the limit of naked eye visibility at +6.5. At opposition, Uranus is visible to the naked eye in dark skies. In the photo we also notice that Regulus is just below the horizon. Let's note here that the orbital period of Neptune, which is the farthest planet from the Sun in our Solar System is roughly 165 years. Counting back 165 years from Menkaure's death (~2500 BC), we come to 2665 BC, which falls in the period Djoser ruled. This means that pyramid building during the III and IV dynasty lasted roughly the time Neptune takes to revolve around the Sun.

10. Summer Solstice Observations in our Epoch

Due to Earth's gradual change in obliquity, the alignment at Summer Solstice of our time of the Sun disk center with the pyramid apex is not that accurate. None the less, the Sun still more or less aligns setting over the Menkaure pyramid. Below we see a photo showing a 3-D presentation of this alignment(satellite image credit: Google Earth/Digital Globe, 2016). On 2016 AD, Summer Solstice took place on June the 20th. Through Cartes du Ciel we see that at time

18:41:24 EET stationed at the incenter, the apparent altitude of the Sun is 2° 51' 7.2" and its azimuth y_p is 295° 39' 37.4". This means that the divergence Δφ' is:

Δφ' = y_P - $φ_{O3}$ = 295° 39' 37" - (296° 19' 21") = - 0° 39' 44" = -0.662°

The angular diameter of the Sun is between 31' 31" and 32' 33". The above difference in azimuth Dφ' is larger than this size. But the present difference between the maximum angular diameter of the Sun and the minimum diameter of the Sun is 1' 2". This not far from the 2501 BC apex - Sun azimuth divergence Δφ (1' 6"). 4,600 years ago the orbital eccentricity of Earth's trajectory was slightly larger than it is today, therefore the difference between the maximum and minimum angular diameter of the Sun would also have been slightly larger. The Sun in our epoch, during Summer Solstice sets slightly to the South of the pyramid apex direction moving gradually behind the pyramid. The Google Earth 3D Buildings feature is a powerful tool we can use to investigate intentional or non-intentional Giza alignments.
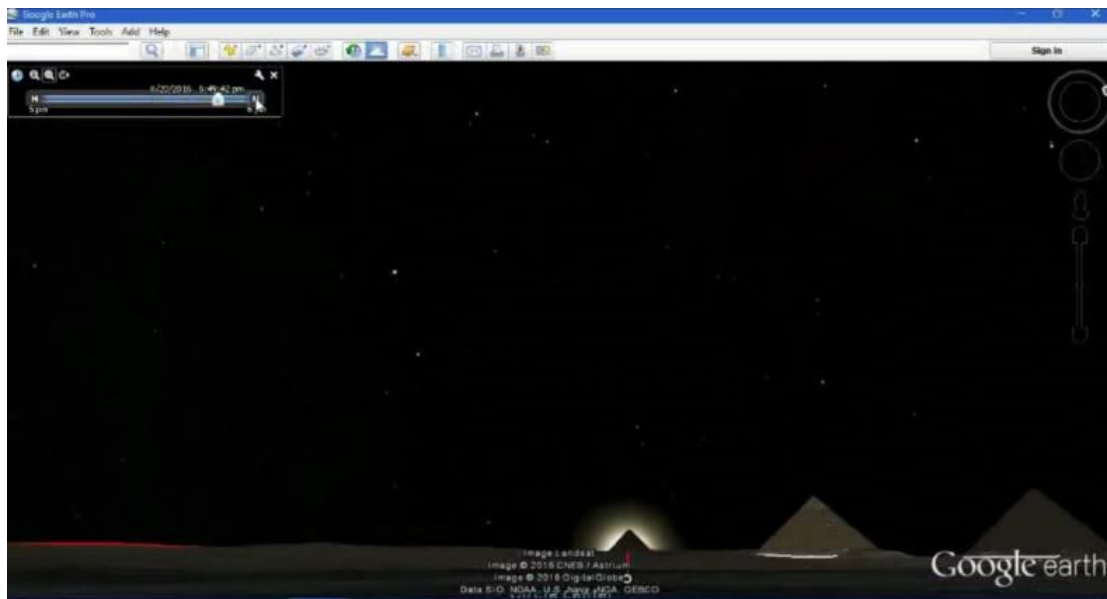
11. Circumcircle center calibration

According to Mark Lehner (GPMP) the exact geographical coordinates of Khufu pyramid's apex are as follows :

29.979201° N, 31.134929° E

This leads to an error in regards to our computed value by:

-0.000018° N, -0.000738° E

We can then compute the calibrated, based on the GPMP value of the center circle position O":
O" = 29.963107 N, 31.150859 E

If somebody visits Giza he can take this value and using his GPS find the observation point just East of the Al Bargasy road.

12. Relations to Ancient Egyptian Religion

Ra or Re is the ancient Egyptian sun god. By the Fifth Dynasty in the 25th and 24th centuries BC, he had become a major god in ancient Egyptian religion, identified primarily with

the noon Sun. In later Egyptian dynastic times, Ra was merged with the god Horus, as Ra-Horakhty ("Ra, who is Horus of the Two Horizons"). He was believed to rule in all parts of the created world: the sky, the earth, and the underworld. He was associated with the falcon or hawk.

To the Egyptians, the sun represented light, warmth, and growth. This made the sun deity very important, as the sun was seen as the ruler of all that he created. The sun disk was either seen as the body or eye of Ra. Ra was the father of Shu and Tefnut, whom he created. Shu was the god of the wind, and Tefnut was the goddess of the rain. Sekhmet was the Eye of Ra and was created by the fire in Ra's eye. She was a violent lioness.

Menkaure's name includes the name of the Sun god Re or Ra. Menkaure literally means "Eternal Like The Souls Of Re". The Summer Solstice alignment might in fact be the reason that Menkaure chose to build a significantly smaller pyramid in regards to his father(Khafre) and grandfather(Khufu). The altitude data of the Giza circumcenter and the Menkaure base need to be computed with accuracy and the center of the circle should be inspected or excavated. Making the Giza Plateau Mapping Project data available to the wide public would surely help Giza pyramid design research. From the Antiquity Now website we copy:

"The summer solstice was especially important in Ancient Egypt because it heralded the coming of Sirius, the brightest star in the night sky. Shortly after Sirius arrived each year, the Nile would overflow its banks and the flood season would begin, which the Egyptians relied on to nourish the land."

13. Conclusions

Given the accuracy of the Giza pyramid triangle incenter Summer Solstice alignment, along with other corroborating evidence, like the Sun - Ra title in Menkaure's name, it does seem that the Giza pyramids were erected based on an original pan-generational geometric blueprint that incorporated, among other things astronomical - stellar alignments.

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*Corrected typos June 27 & July 2, 2016